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An MHD flow of non-Newtonian fluids with CNTs and heat transfer across a linearly shrinking sheet with slip and Biot number

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ABSTRACT

The present paper focuses on electrically driven magnetohydrodynamic non-Newtonian fluid flow (viscoelastic fluid)with CNTs and heat transmission across a linearly shrinking sheet. The flow is urged with the partial slip conditions, radiation and heat source/sink. Single walled and multi walled CNTs are considered in this analysis. An exact solution is extracted for the governing equations of the aforesaid flow of two types of non-Newtonian fluids like second grade fluid and Walters' liquid B with CNTs using similarity transformations. The flow velocity is pretentious due to many physical parameters like magnetic force, visco-elasticity, slip parameter, whereas the energy profile is pretentious due to Prandtl number, Biot number, heat sink/source parameter and radiation parameter. The existence of unique and dual solutions domains are traced by combining the above parameters and velocity, energy profiles and skin friction are explained using graphs. The results reveal that the dual solutions exist for both types of viscoelastic fluids under specific flow restrictions. In particular, dual solutions exist for second grade fluid for both slip and no-slip restrictions. And similar observations are found for Walters' liquid B fluid for unique slip flow when viscoelastic parameter value increases above a definite value. Inclusion of magnetic field terminates the ambiguity of existence of solution for different λ_1 values for both viscoelastic fluids.

1. Introduction

It is well known that majority of the liquids are non-Newtonian liquids which are profusely utilized in many manufacturing processes in industries, like heat exchangers, geothermal systems, cooling of electrical components, nuclear reactors, crystal formation, cosmetic products, metallurgical operations and several others. This is because non-Newtonian fluids finds more advantageous in less heat transfer rate and requires less power in stretching/shrinking a sheet when compared to Newtonian fluids. The non-Newtonian fluids like second grade fluid and Walters' liquid B are focused in this study. In 1950s, Rivlin and Ericksen[1] proposed the fundamental model for second grade fluid and reported the existence of the exact solution. Later this model was used by many other authors employing different analytical techniques and explained the exact solution (ES) of flow of second grade fluid. Bandelli and Rajagopal [2] focusedat the ES of second-grade fluids flow employing the integral transform procedure. Mahmood *et al.* [3] identified ESs for the oscillatory flow of modified second-grade fluid between two cylinders. Several other authors [4–8] had credited the investigated results of second grade fluid flow to its account. In addition to this, we take into account Walter's liquid model-B, which forecasts the characteristics of various polymeric fluids that emerge in chemical and bioengineering fields. This Walters' liquid model-B was first suggested by Walter [9]. Later, Munirathinam *et al.* [10] explored the results of Walters' liquid B flow towards a penetrable stretching surface with ohmic effect and inclined magnetic force is applied on the flow. Analytical solutions are extracted for fractional Walters' liquid B flow across an oscillating plate, by Qaseem *et al.* [11]. Pandya *et al.* [12], Damala *et al.* [13] and Sneha *et al.* [14] also reported on the viscoelastic Walters' liquid B flow. All the above studies are restricted to viscoelastic fluids. The succeeding paragraph specifies the significance of using carbon nanotubes filled with water.

A tube made up of carbon atoms with a diameter commonly measured in nanometers is referred to as a carbon nanotube (CNT).CNTs exhibit incredible electrical [15,16] and thermal conductivity [17,18] due to their nanostructure and strong bonding between the carbon atoms. Due to the numerous potential uses, including medicine administration, ion transport, water purification and desalination, carbon nanotube (CNT) are used as an additives in the working fluid(base fluid) as part of the passive method to enhance heat transmission. This amplifies the interest of the researchers in investigating the CNT nanofluid flow and heat transfer problems. So far, many papers have competently reviewed the performance of CNT nanofluid. Kipper and Silva [19] explained the temperature and porosity effects when water flows through carbon nanotube layers. Saleh *et al.* [20] intimated the medical applications of the use of CNTs in a convective flow. Mahabaleshwar *et al.* [21] reported on the approximation of the Cattaneo-Christo on the viscoelastic flow with CNTs and heat transfer. Thiemann *et al.* [22] conducted an experiment of water flow across a single wall nanotubes. This experiment was between carbon nanotubes and boron nitride nanotubes. The results reveal that there is an ultrafast water transport in carbon nanotubes when regulated by oxygen with 5 time's lower friction whereas larger friction on boron nitride occurs due to nitrogen–hydrogen molecules interactions.

Furthermore, BLF of NN fluid across a SS/CS has fascinated the interest of modern researchers owing to its growing applicability in numerous engineering processes. Crane [23] was the first to discuss the flow over stretching sheet(SS) while Wang [24] was the first author to discuss the flow on a contracting sheet (CS). Later, Miklavcic and Wang [25], Nadeem and Awais [26], Fang and Zhang [27] and many other

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Nomenclature

B	Constant (–)
B_0	Magnetic force (–)
Bt	Biot number (–)
C_p	Specific heat capacity (J Kg^{-1} K^{-1})
C_f	Skin friction coefficient (–)
F	Similarity function (–)
H	Heat transfer coefficient (–)
H	Hypergeometric function (–)
I	Heat source/sink parameter (–)
k_1	Viscoelastic parameter (NA^{-2})
l	Characteristic length (m)
L	Lorentz force constraint (Wbm^{-2})
Nu_x	Nusselt number (–)
p	Pressure (Pa)
Pr	Prandtl number (–)
q_r	Radiative flux (Wm^{-2})
Q_0	Heat absorption/ generation (–)
R	Radiation parameter (–)
Re_x	Reynolds number (–)
T	Temperature (K)
(u,v)	Components of velocity in x- and y-direction (ms^{-1})
(x,y)	Co-ordinate axis (m)

Greek symbols

β^*	Constant (–)
K	Thermal conductivity (WKg^{-1} K^{-1})
η	Similarity variable (–)
μ	Dynamic viscosity (Kgm^{-1} s^{-1})
ρ	Density (Kg m^{-3})
σ	Electrical conductivity (Sm^{-1})
ν	Kinematic viscosity ($m2s^{-1}$)
ψ	Stream function (–)
λ	Slip parameter (–)
ξ^*	Permeability (Hm^{-1})

Subscripts

nf	Nanofluid (–)

Abbreviations

CNT	Carbon nanofluid (–)
SWCNT	Single walled carbon nanotubes (–)
MWCNT	Multi walled carbon nanotubes (–)
MHD	Magnetohydrodynamics (–)
BLF	Boundary layer flow (–)
b.cs.	Boundary conditions (–)
ES	Exact solution (–)
NN	Non-Newtonian (–)

authors had explained the importance of flow over a SS/CS. In addition to this, the effectiveness of the BLF is enhanced by the inclusion of many physical parameters like, magnetic effect, radiation effect, slip effect, heat source/sink, many more, which strongly influences the heat transfer properties. This seeks the applications in industrial operations like, metal thinning, heat exchangers, electronic chips, etc. Midya [28] studied the heat transmit in an MHD viscous flow on a linearly penetrable CS. Vishalakshi *et al.* [29,30] discussed the impact of magnetic force on NN fluid flow and heat transmission with slip condition over an absorbent SS and CS. Anusha *et al.* [31] investigatedan MHD nanofluid flow past an extending/contracting penetrable plate with Brinkman ratio. The nanofluid flow across a SS/CS with radiation impact and mass transpiration had been studied by many authors [32–35]. The ES of various MHD flow past a SS/CS has been reported by Hayat *et al.* [36]. Wahid *et al.* [37] explained MHD effect on Casson nanofluid flow over a SS and also reported the heat transfer rate. Animasaun*et al.* [38] explored the impacts of magnetic flux on tri-hybrid nanofluid flow with heat source or sink on a heated surface. Bhatti *et al.* [39,40] investigated nanofluid and hybrid nanofluid flow over a flat surface with different geometries. This investigation reveals the importance of MHD effect and thermal slips. Turkyilmazoglu [41] derived algebraic solutions for nanofluids flow and energy transfer rate over a permeable surfaces. (See Table 1Table 2)

With the motivation of the above research and applications of the BLF, the present work concentrate to study the velocity and thermal properties of two types of viscoelastic fluids (non-Newtonian fluid) like second grade fluid and Walters' liquid B flow with water based CNTs on a SS with magnetic impact, radiation, heat sink/source and also partial slip effect. Dual and unique solutions are obtained which is dependent on many physical parameters. Deviations in skin friction and Nusselt number are interpreted using graphical forms. The inclusion of water-based CNTs with an external magnetic effect and partial slip affecting the velocity field and radiation effect, as well as a heat source or sink affecting the temperature field, distinguishes this study.

Table 1
Physical quantities of H2O and CNTs [Sneha [48], Sreenivasulu [49]].

Physical quantities	H_2O	*SWCNT*	*MWCNT*
$C_p(J/KgK)$	$41{,}790 \times 10^{-1}$	4.25×10^{2}	7.96×10^{2}
$\rho(Kg/m^3)$	9971×10^{-1}	26×10^{2}	16×10^{2}
$\kappa(W/mK)$	613×10^{-3}	66×10^{2}	30×10^{2}
$\sigma(\Omega/m)^{-1}$	5×10^{-2}	48×10^{6}	38×10^{6}

2. Formulation of the flow problem

Fig. 1 describes the steady, electrically conducting non-Newtonian laminar flow of Walters' liquid B fluid and second grade fluid flow

Table 2
Comparison of the present work against published works.

Published works with Authors name	Fluids	Value of β
Present work	Non-Newtonian + CNTs	$A_1\lambda\beta^{*3} + (A_1 - A_2k_1)\beta^{*2} - A_3L\lambda\beta^{*} - (A_3L - A_2) = 0.$
Anil *et al.* [50]	Non-Newtonian	$\lambda\mu^{*3} + (1-\alpha_\nu)\mu^{*2} - M\lambda_1\mu^{*} - M + 1 = 0.$
Bataller [42]	Non-Newtonian	$\lambda\alpha^3 + (1-\beta)\alpha^2 - \delta\alpha + 1 = 0.$
Cortell [51]	Non-Newtonian	$\lambda_1Rr^3 + (1+\lambda_1)r^2 - Rr - M - 1 = 0.$

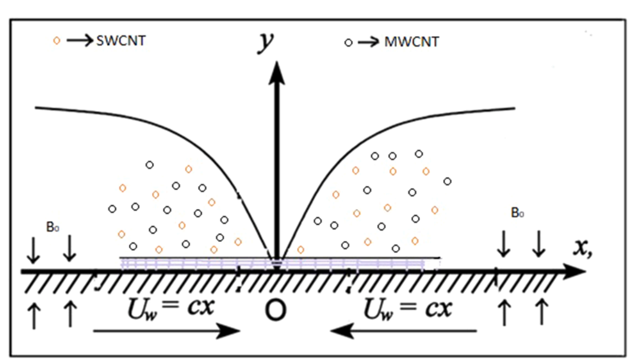

Fig. 1. Problem representation.

with CNTs on a SS. The path of the SS is measured along x-axis while the fluid is allowed to flow along y-axis.

The magnetic field B_0 is employed normal to the SS. $u_w(x) = bx$ and T_w is the velocity and temperature of the SS, respectively. The partial slip condition is assumed during the flow on the shrinking sheet. The radiation effects and heat generation/absorption is included in the temperature equation. With all the above considerations, the flow problem is modeled, and the resulting governing equations are as follows: (see Bataller [42], Anil *et al.* [43] and Turkyilmazoglu [44,45])

$$\frac{\partial u}{\partial x}+\frac{\partial \mathrm{v}}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+\mathrm{v}\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}\pm\xi^*\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+\mathrm{v}\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x\partial y}\right\}-\frac{\sigma_{nf}B_0^2}{\rho_{nf}}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+\mathrm{v}\frac{\partial T}{\partial y}=\left(\kappa_{nf}\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}+Q_0(T-T_w)\right)\frac{1}{(\rho C_p)_{nf}}. \tag{3}$$

Enforced b.cs. are:

$$u(x,0)=-u_w(x)+l\frac{\partial u}{\partial y},\quad \mathrm{v}(x,0)=0,\quad \kappa_{nf}\left.\frac{\partial T}{\partial y}\right|_{y=0}=h(T_w-T), \tag{4}$$

$$u(x,\infty)=0,\quad \frac{\partial u}{\partial y}=0,\quad T(x,\infty)=T_\infty. \tag{5}$$

Here, u and v are the corresponding x- and y-direction velocity elements of the flow, respectively. The indication of positive sign refers to second grade fluid ('+' sign) and negative sign refers to and Walters' liquid B fluid ('-' sign) in the RHS of Eq.(2). ξ^* constitutes the viscoelasticity of the fluid. Remaining parameters are listed in the nomenclature.

(i) Approximation of radiative heat flux (q_r)

Using Rosseland approximations, the term q_r is simplified as below:

$$q_r=-\frac{4\omega^*}{3\upsilon^*}\frac{\partial T^4}{\partial y} \tag{6}$$

where, υ^* represents average absorption factor and ω^* refers Stefan Boltzmann constant. The temperature variation arising in the flow is assumed such way that, the term T^4 is expanded as a linear function of the temperature. Hence, the term T^4 is expanded using Taylor's series expansion and terms of higher order are ignored, resulting to

$$T^4\cong T_\infty^3(-3T_\infty+4T) \tag{7}$$

Therefore,

$$\frac{\partial q_r}{\partial y}=-\frac{16\omega^* T_\infty^3}{3\upsilon^*}\frac{\partial^2 T}{\partial y^2} \tag{8}$$

Using Eq.(6) and Eq.(8), Eq.(3) is rewritten as:

$$u\frac{\partial T}{\partial x}+\mathrm{v}\frac{\partial T}{\partial y}=\left(\frac{\kappa_{nf}}{(\rho C_p)_{nf}}+\frac{16\omega^* T_\infty^3}{3\upsilon^*(\rho C_p)_{nf}}\right)\frac{\partial^2 T}{\partial y^2}+\frac{Q_0(T-T_\infty)}{(\rho C_p)_{nf}}. \tag{9}$$

(ii) Thermal properties of nanofluid

The effective thermophysical models of nanofluid are **(a)** density ρ_{nf}, **(b)** dynamic viscosity μ_{nf}, **(c)** thermal conductivity κ_{nf}, **(d)** heat capacity $(\rho C_p)_{nf}$ and **(e)** electrical conductivity σ_{nf} are considered as below (refer Hayat *et al.* [46], Anuar *et al.* [47]):

(a) $\rho_{nf}=(1-\phi)\rho_{H_2O}+\phi\rho_{CNT}$,

(b) $\mu_{nf}=\frac{1}{(1-\phi)^{2.5}}$,

(c) $\kappa_{nf}=\kappa_{H_2O}\left[\dfrac{1-\phi+2\phi\frac{\kappa_{CNT}}{\kappa_{CNT}-\kappa_{H_2O}}\ln\frac{\kappa_{CNT}+\kappa_{H_2O}}{2\kappa_{H_2O}}}{1-\phi+2\phi\frac{\kappa_{H_2O}}{\kappa_{CNT}-\kappa_{H_2O}}\ln\frac{\kappa_{CNT}+\kappa_{H_2O}}{2\kappa_{H_2O}}}\right]$,

(d) $(\rho C_p)_{nf}=(1-\phi)(\rho C_p)_{H_2O}+\phi(\rho C_p)_{CNT}$.

(e) $\sigma_{nf}=\left[1+\dfrac{3(\sigma_{CNT}-\sigma_{H_2O})\phi}{(\sigma_{CNT}+2\sigma_{H_2O})-(\sigma_{CNT}-\sigma_{H_2O})\phi}\right]\sigma_{H_2O}$

(iii) Similarity analysis and solution

The stream function ψ and the similarity variable

$$\psi=\sqrt{\nu xU_w}f(\eta),\eta=y\sqrt{\frac{U_w}{x\nu}}and\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{10}$$

where, ψ denotes the stream function having $u=\psi_y=cxf'(\eta)$&$\mathrm{v}=-\psi_x=-\sqrt{\nu c}f(\eta)$ with the variable η.

$$A_1f_{\eta\eta\eta}-A_2\left[f_\eta^2-ff_{\eta\eta}\right]+k_1A_2\left[2f_\eta f_{\eta\eta\eta}-ff_{\eta\eta\eta\eta}-(f_{\eta\eta})^2\right]-A_3Lf_\eta=0, \tag{11}$$

$$(C_1+R)\theta_{\eta\eta}+C_2Prf\theta_\eta+PrI\,\theta=0. \tag{12}$$

With the imposed b.cs. as

$$f_\eta(0)=-1+\lambda_1f_{\eta\eta}(0),f(0)=0,f_\eta(\infty)=0 \tag{13}$$

$$\theta_\eta(0)=-\frac{Bt}{C_1}[1-\theta(0)],\theta(\infty)=0. \tag{14}$$

where, $k_1=\pm\frac{c\,\xi^*}{\nu}$ is viscoelasticity parameter in the accompany of $k_1>0$ for second-grade fluid and $k_1<0$ corresponds to Waters' liquid B fluid, $\lambda=l\sqrt{\frac{c}{\nu}}$ is slip parameter, $R=\frac{16\,\omega^* T_\infty^3}{3\,\upsilon^*\kappa}$ is thermal radiation parameter, Lorentz force parameter is denoted as $L=\frac{\sigma B_0^2}{\rho c}$, $Pr=\frac{\kappa}{(\rho C_p)\nu}$ is known as Prandtl number, Biot number is defined as $Bt=\frac{h}{\kappa}\sqrt{\frac{\nu}{c}}A_1=\frac{\mu_{nf}}{\mu_{H_2O}}$ $A_2=\frac{\rho_{nf}}{\rho_{H_2O}}$ $A_3=\frac{\sigma_{nf}}{\sigma_{H_2O}}$ $C_1=\frac{\kappa_{nf}}{\kappa_{H_2O}}$ and $C_2=\frac{(\rho C_p)_{nf}}{(\rho C_p)_{H_2O}}$.

3. Exact solution analysis for velocity equation

The exact analytical solutions for the Eq. (11) is obtained by utilizing the Eq. (10) with related to boundary conditions Eqs. (13), is considered as follows

$$f(\eta)=\frac{-1}{\beta^*(1+\lambda\beta^*)}[1-exp(-\beta^*\eta)], \tag{15}$$

Upon substitution of Eq. (10) into Eq. (11), we obtain the following algebraic equation

$$A_1\lambda\beta^{*3}+(A_1-A_2k_1)\beta^{*2}-A_3L\lambda\beta^*-(A_3L-A_2)=0. \tag{16}$$

To find the roots of Eq.(16), we rewrite the above cubic equation in terms of incomplete cubic equation as: $\tau^{*3}+p\tau^*+q=0$, (17)where $\tau^*=\mu^*+\frac{(A_1-k_1A_2)}{3\lambda A_1}$,$r=\frac{-3\lambda^2A_1A_3L-(A_1-k_1A_2)^2}{3\lambda^2A_1^2}$ and

$$s=\frac{2(A_1-k_1A_2)^3+9\lambda^2A_1A_3L(A_1-k_1A_2)-27\lambda^2A_1^2(A_3L-A_2)}{27\lambda^3A_1^3} \tag{18}$$

The following are the roots of the incomplete cubic equation:

$$\tau^*=a_1+a_2\ ;\ \ \tau_{2,3}^*=-\frac{1}{2}(a_1+a_2)\pm i\frac{\sqrt{3}}{2}(a_1-a_2),$$

where, $a_1=\sqrt[3]{-\frac{q}{2}+\sqrt{a_3}}$, $a_2=\sqrt[3]{-\frac{q}{2}-\sqrt{a_3}}$ and $a_3=\left(\frac{p}{3}\right)^3+\left(\frac{q}{2}\right)^2$.

Note that when no slip condition is applied for an MHD boundary layer flow i.e., $\lambda=0$ in Eq. (16), results $\beta^*=\sqrt{(A_3L-A_2)/(A_1-k_1A_2)}$ and the corresponding similarity solution as below,

$$f(\eta) = \frac{exp\left[-\sqrt{(A_3L-A_2)/(A_1-k_1A_2)}\right]-1}{\sqrt{(A_3L-A_2)/(A_1-k_1A_2)}}$$

Therefore, due to no slip condition, unique solution exists for second grade fluid for only two cases: (i) $k_1 > 1$ when $L < 1$ and (ii) $k_1 < 1$ when $L > 1$. But in the case of Walters' liquid B, only unique solution exists for $k_1 < 0$ when $L > 1$.

4. Exact solution for thermal equation

We introduce a new variable $\xi = \frac{Pr}{\beta^{*2}}e^{-\beta\eta}$ to transform the Eq. (12) to the following equation

$$\xi\,\Theta_{\xi\xi}(\xi) + (1+m-n\,\xi)\Theta_{\xi}(\xi) + \frac{p}{\xi}\Theta(\xi) = 0, \tag{17}$$

where, $m = \frac{C_2 \mathrm{Pr}}{\beta^{*2}(C_1+R)(1+\lambda\beta^{*})}, n = \frac{C_2}{\beta^{*2}(C_1+R)(1+\lambda\beta^{*})}, p = \frac{I\,\mathrm{Pr}}{\beta^{*2}(C_1+R)}$ with boundary conditions

$$\Theta_{\xi}\left(\xi = \frac{Pr}{\beta^{*2}}\right) = -\frac{Bt}{C_1}\left[1-\theta\left(\xi = \frac{Pr}{\beta^{*2}}\right)\right], \Theta(\xi=0) = 0, \tag{18}$$

The thermal equation is solved by using Eq. (17) along with Eq. (18) and obtained the solution in terms of Hypergeometric functions.

$$\Theta(\eta) = a\,\xi^{z}\,H\left[\frac{q_1+q_2}{2},\ q_2+1,\ n\xi\right]. \tag{19}$$

a being the integration constant which is evaluated using b.cs. in Eq. (18):

$$a = \frac{Bi}{C_1\left\{\begin{array}{c}\frac{Bi}{C_1}p^{\frac{q_1+q_2}{2}}H\left[\frac{q_1+q_2}{2},\ q_2+1,\ np\right] + \left(\frac{q_1+q_2}{2}\right)p\beta H\left[\frac{q_1+q_2}{2},\ q_2+1,\ np\right] \\ +\ \frac{n\beta}{1+q_2}\left(\frac{q_1+q_2}{2}\right)H\left[1+\frac{q_1+q_2}{2},\ q_2+2,\ np\right]\end{array}\right\}} \tag{20}$$

5. Quantities of attentiveness

(a) Skin friction factor C_f is defined as,

$$C_f = \frac{2\tau_w}{\rho_{nf}U_w^2},$$

Where, $\tau_w = \rho_{nf}[\nu_{nf}(\frac{\partial u}{\partial y}) \pm \xi^{*}(2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + u\frac{\partial^2 u}{\partial x \partial y})]_{y=0}$.

Therefore, C_f standardized to $\frac{1}{2}(Re_x)^{-1/2}C_f = \left[\frac{A_1}{A_2} - 3k_1\{1-\lambda f_{\eta\eta}(0)\}\right]f_{\eta\eta}(0)$.

(b) The definition of Nusselt number is taken as,

$$Nu_x = \frac{xq_w}{\kappa_f(T_w - T_\infty)},$$

where, $q_w = -(\kappa_{nf}+R)\frac{\partial T}{\partial y}\Big|_{y=0}$.

Therefore, $Re_x^{1/2}Nu_x = -(C_1+R)\theta_\eta(0)$.

6. Analytical results and explanation

Investigation of the impact of magnetic field and partial slip on an NN fluids flow with CNTs over a SS is analyzed and presented in graphical forms. Here two types of NN fluids are considered such as second grade fluid and Walters' liquid B. Also, fluid flow investigation deals with two types of carbon nanotubes i.e., SWCNT and MWCNT with the volume fraction $\phi = 0.1$. For two different values of β^{*} of Eq.(16) for different values of viscoelastic parameter (k_1), magnetic parameter (L) and slip parameter (λ_1) double similarity solution exist for second grade fluid $(k_1 > 0)$ for slip value $\lambda_1 = 0.5$. It is also noted that no solution exist for $k_1 = 0$..

Figs. 2-4 represents the significance of the parameters such as (k_1), (L) and (λ_1) on the velocity profile of the NN fluids with CNTs. Fig. 2(a) and 2(b) represents the impact of k_1 on the second grade fluid at the values $L = 0.5$, $\lambda_1 = 0.5$ and $L = 0.5$, $\lambda_1 = 7$ respectively. It is observed that, due to the dominance of elastic behavior over viscosity, the first solution for second-grade fluid is increased while the second solution is found to decrease for higher slip values. The influence of SWCNT and MWCNT can also be observed in this presentation. With the increasing k_1 values, for the first solution, the BL thickness is reduced for SWCNT flow when compared to MWCNT flow. But for second solution, boundary thickness is reduced due to MWCNT. Furthermore, the magnetic field opposes the phenomena of transport, which is illustrated in Fig. 3(a) and 3(b),as a reduction in the thickness of the BL for the first solution and a comparable impact on boundary slip is presented in Fig. 4(a) and 4(b). Consequences of magnetic field parameter on velocity profile explain that the velocity decreases for both SWCNTs and MWCNTs for higher values of magnetic field. It is also investigated that boundary layer thickness reduces for higher magnetic field.

Temperature profiles of second grade fluid are displayed in the Figs. 5-7 for various values of Biot number (Bt), radiation parameter (R), and heat absorption parameter (I). Temperature of the flow is noted as increasing for increased values of R and Bt and, decreasing for increased I values for both first and second solution. With growing values of R, Bt and I the thermal enhancement is higher due to MWCNT when compared to SWCNT. Generally larger values of R produce extra heat to operating liquid that shows associate improvements within temperature profile.

Skin friction structure for different L and k_1 values are displayed in Figs. 8-10. For $k_1 = -0.2$, the effect of the magnetic field on the range of solutions and the uniqueness of solutions is explored in Fig. 8. It portrays that for Walters' liquid B flow, intense MHD impact is destroying the ambiguity of existence of the similarity solution and confirms for unique solution for slip and no slip flows. Fig. 9 confirms that for second grade fluid, it is important to note that the presence of an adequately strong magnetic field stabilizes the ambiguous situation around the existence of a solution and leads to a singularly comparable solution for slip flow over a SS. As a result, the flow velocity caused by the sheet's contraction can be controlled by the magnetic field.

Fig. 10 demonstrates the skin friction structure for various values of k_1. It reveals that the solution range of k_1 is extended as a result of the fluid's elastic property's supremacy over its viscous property. Additionally, if k_1 increases then double solution exist throughout the entire region, i.e., for every $k_1 < 0$ value, double solutions appear to exist. This novel finding occurs for $L = 0.5$.

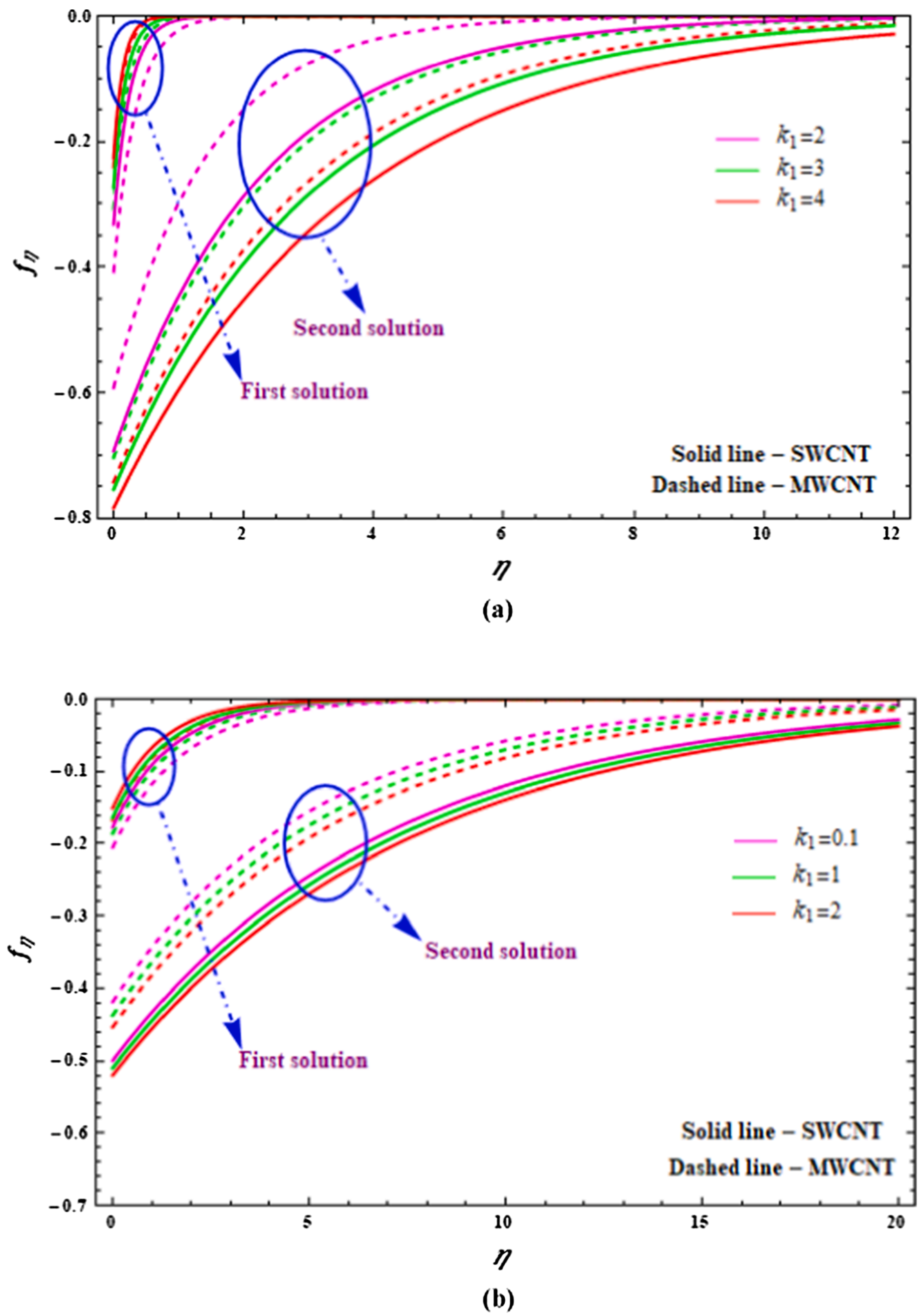

Fig. 2. Presentation of the axial velocity of the second grade fluid flow for a range of values of k_1 with (a)$L = 0.5,\ \lambda_1 = 0.5$ and (b)$L = 0.5,\ \lambda_1 = 7$.

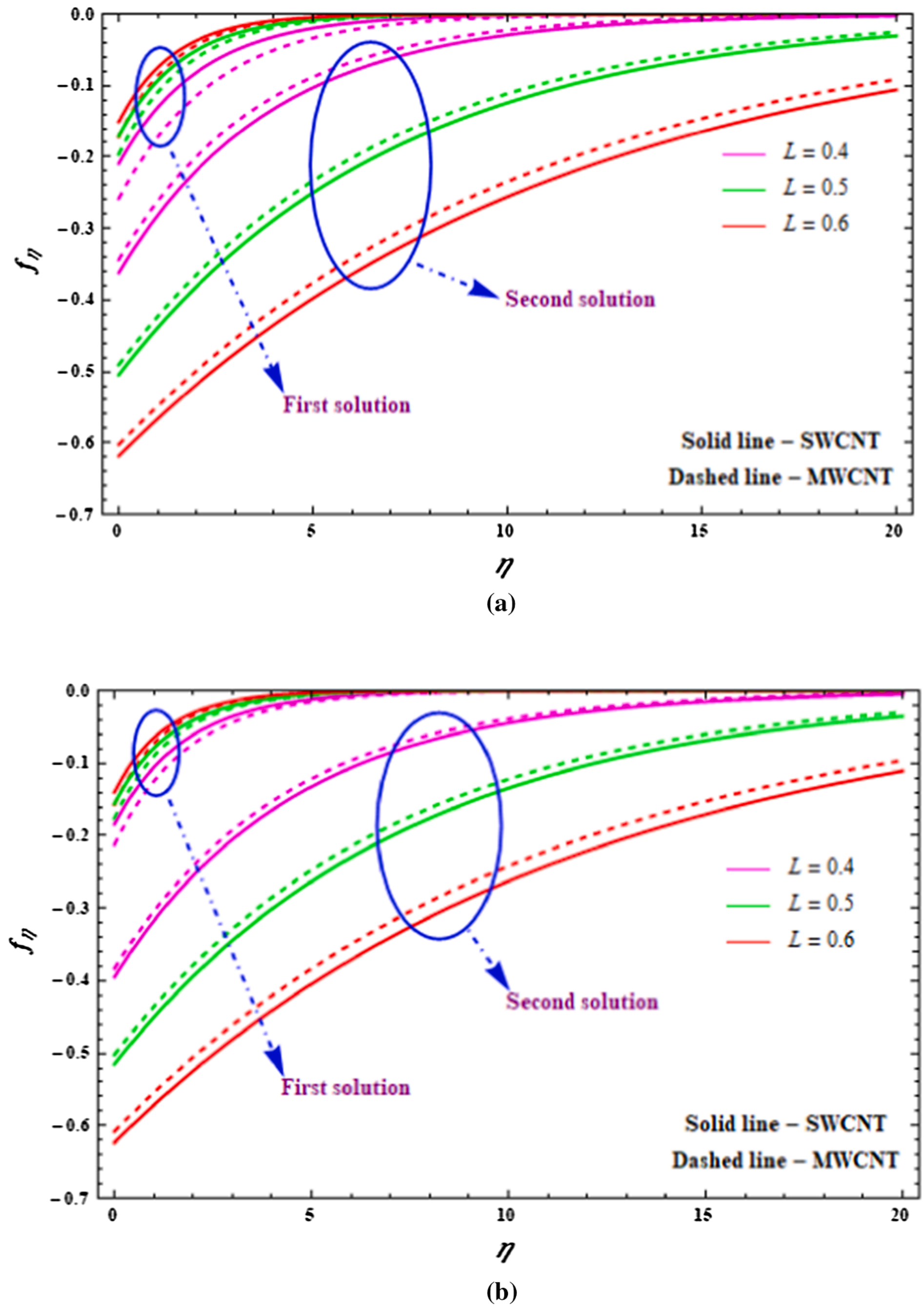

Fig. 3. Presentation of the axial velocity of the second grade fluid flow for a range of values of L with (a)$k_1 = 0.5,\ \lambda_1 = 6$ and (b)$L = 1.5,\ \lambda_1 = 6$..

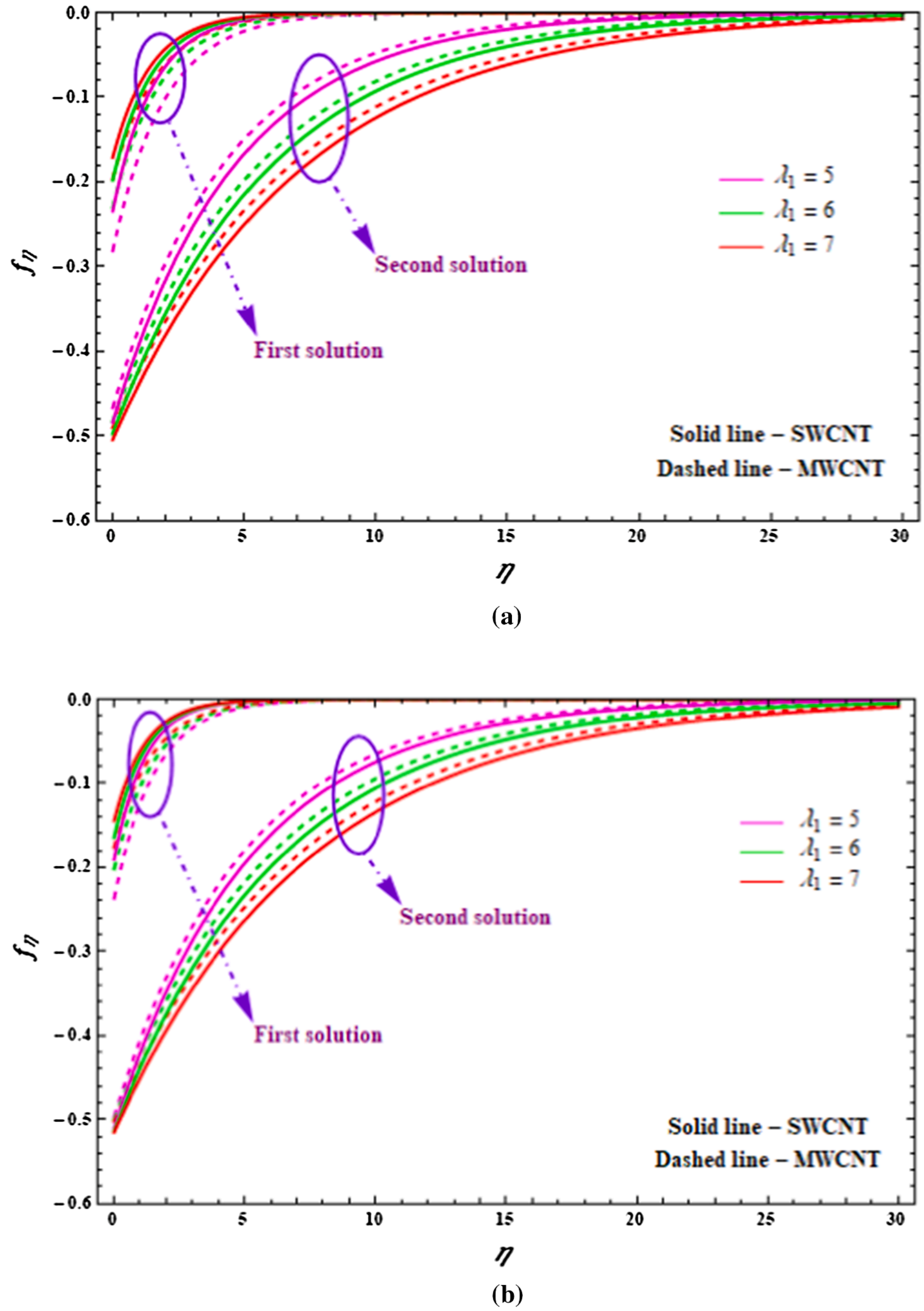

Fig. 4. Presentation of the axial velocity of the second grade fluid flow for a range of values of λ_1 with (a)$k_1 = 0.5,\ L = 0.5$ and (b)$k_1 = 1.5,\ L = 0.5$.

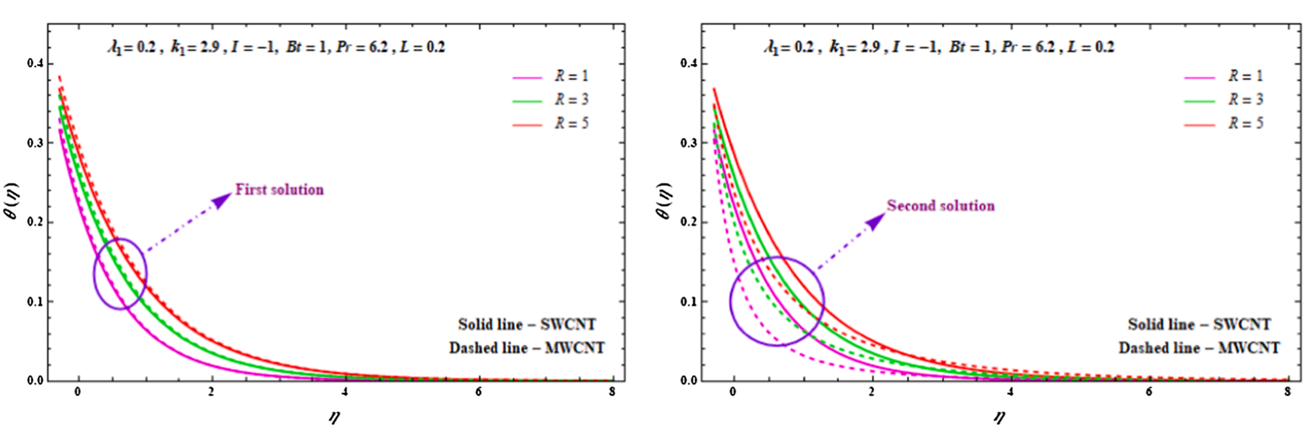

Fig. 5. Heat transport phenomena for a range of values of R.

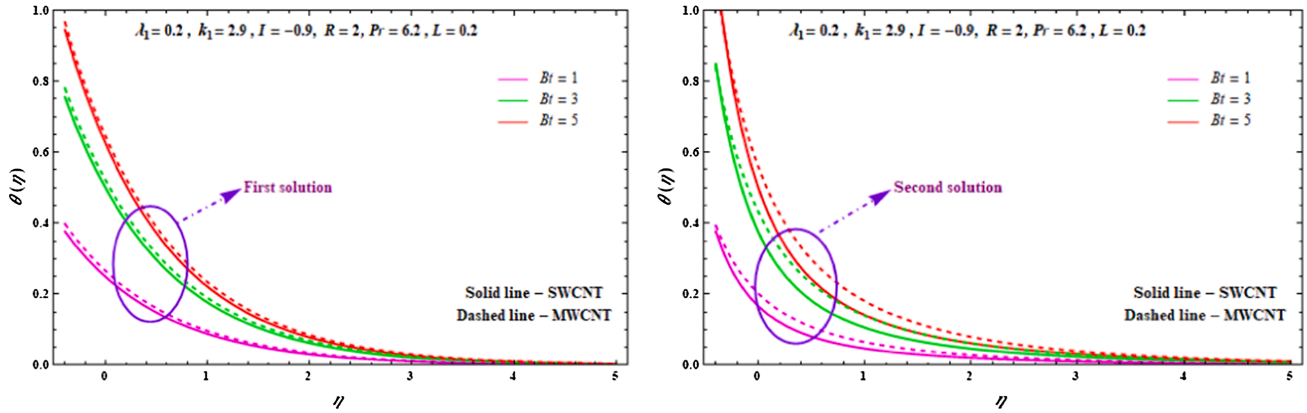

Fig. 6. Heat transport phenomena for a range of values of Bt.

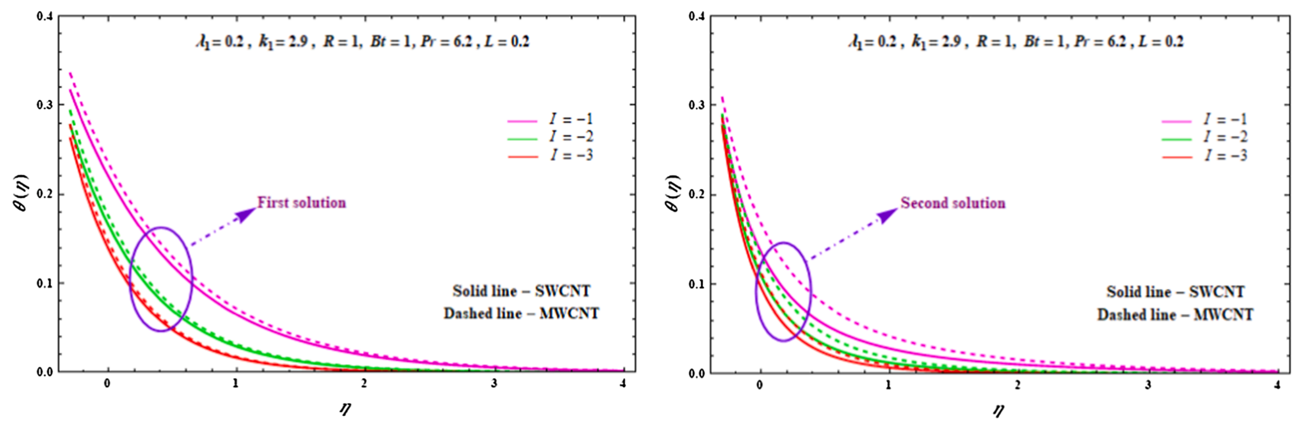

Fig. 7. Heat transport phenomena for a range of values of I.

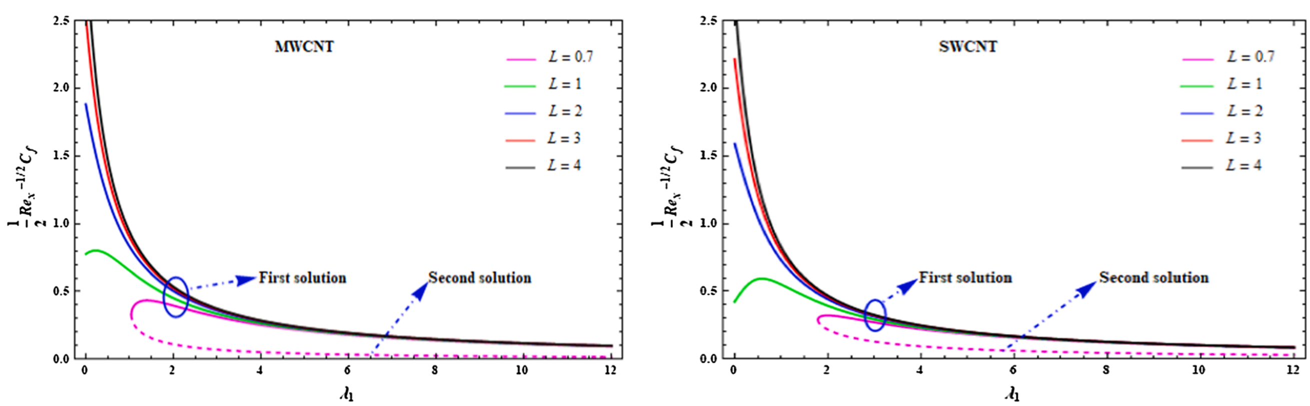

Fig. 8. Skin friction structure for a range of values of L with $k_1 = -0.2$, Walters' liquid B fluid flow ($k_1 < 0$).

7. Conclusion

The BLF of non-Newtonian fluid with CNTs which is impacted by slip and the magnetic effect is analyzed and heat transmission enhancement is also noted. The following are the remarks made on the present study.

- For a particular set of values of the relevant parameters, double solutions exist forboth the viscoelastic fluids.
- For second grade fluid ($k_1 > 0$), double solution exist due to slip.
- The solution presence range of k_1 is increased if elastic property of second grade fluid predominates over viscous property.
- Finally, inclusion of magnetic field terminates the ambiguity of existence of solution for different λ_1 values for both viscoelastic fluids.
- MWCNT impact is more superior compared to SWCNT.

7.1. Validation

The temperature behavior of the present work is verified with the closed form solution proposed by Bataller [42] and Anil *et al.* [50]. Furthermore, the concentration behavior of the present work is verified with the closed form solution proposed by Cortell [51].

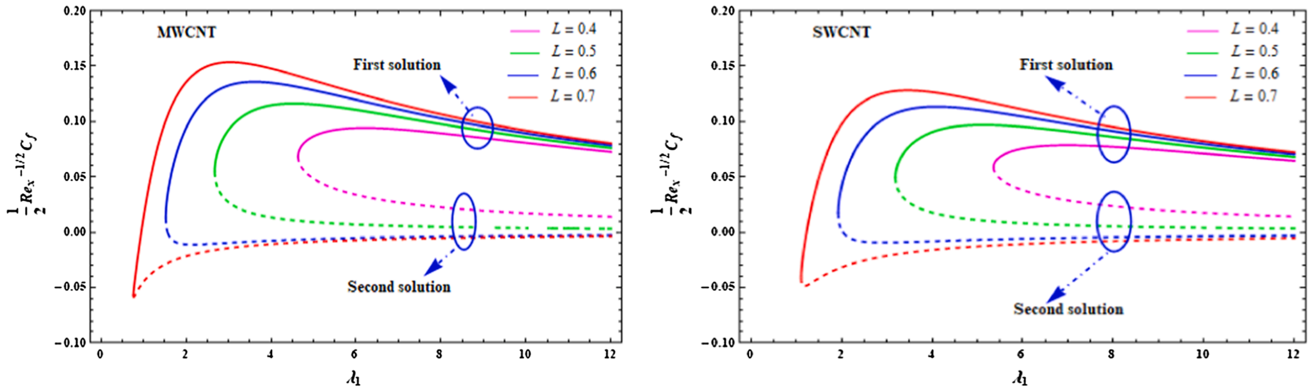

Fig. 9. Skin friction structure for a range of values of L with $k_1 = 0.5$, Second grade fluid flow ($k_1 > 0$).

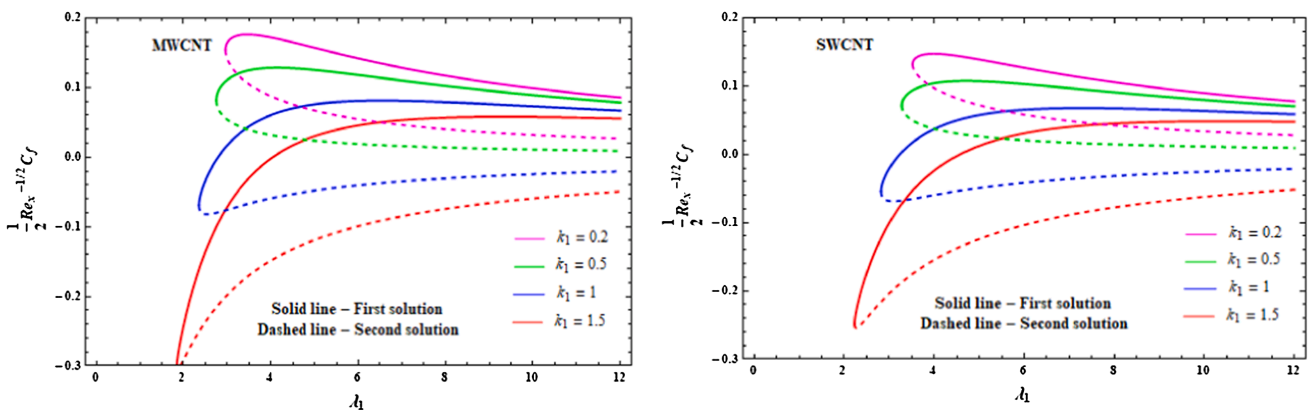

Fig. 10. Skin friction structure for various values of k_1 for second grade fluid ($k_1 > 0$).

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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